Ranking Alternative Warehouse Area Assignments: A Multiattribute Approach

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Abstract: The paper considers a choice problem among alternative space allocations. The choice is often made intuitively because of the difficulty in simultaneously considering multiple criteria (i.e., space categories), some of which are of conflicting nature. The paper employs the methodology of multiattribute value functions to score and rank seven suggested area assignments. Through explicit consideration of trade-offs among the four space categories, the methodology yields a rank ordering which is consistent with the decision maker's preferences. In addition to ranking the alternatives, the formal analysis provided important insight into the trade-off and value structure over the space attributes to facilitate a new area assignment which was superior to the ones originally proposed.

▪ Industrial engineers and managers are often faced with a problem of choosing among several alternative area assignments. Planning is performed by the industrial engineers, and the final choice usually rests with some level of management. The final decision is often based upon the engineer's recommendations, and is frequently made on an intuitive basis partially because of lack of a formal framework to systematically and consistently evaluate these factors or the trade-offs among them.

This paper considers an actual problem of choosing among several area assignments for a metal cutting warehouse. It presents a framework that enables a systematic comparison of the alternative assignments, yielding a choice that is consistent with the decision maker's preferences and values. In our specific application, the formal analysis not only rank ordered the various alternatives but also provided enough insight into the various attributes and trade-offs to facilitate a new area assignment that was superior to the ones originally designed by the industrial engineers. The paper focuses on space assignment and not on the actual layout, although alternative area assignments were presented in layout form.

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The presented methodology of multiattributed value functions is general and can be applied to any setting where multiattributed alternatives are to be ranked and evaluated.

The Problem Area

The firm considered in this paper consists of three sections: (1) workshop, (2) metal cutting, and (3) assembly. Figure 1 presents a schematic representation of the organizational structure. The metal cutting section consists of advanced metal cutting machinery where the actual cutting is performed by specific tools that are hooked onto the machines. The tools department consists of a warehouse and a plant (tool preparation) responsible for maintaining the tools in satisfactory working conditions. The maintenance section of tool preparation draws its tools from the metal cutting warehouse whose layout is the focus of this paper. The arrows in Fig. 1 indicate the possible flow of tools between various departments.

The warehouse is a 10-m × 12-m rectangle with a large opening in the south side (suitable for passage of forklifts) and a smaller opening on the northern side. The area assignment for the warehouse is an allocation problem involving the following four competing activities:

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A. Storage space for heavy parts
B. Shelf storage space (superslot)
C. Cabinet storage (Vidmar)
D. Office space.

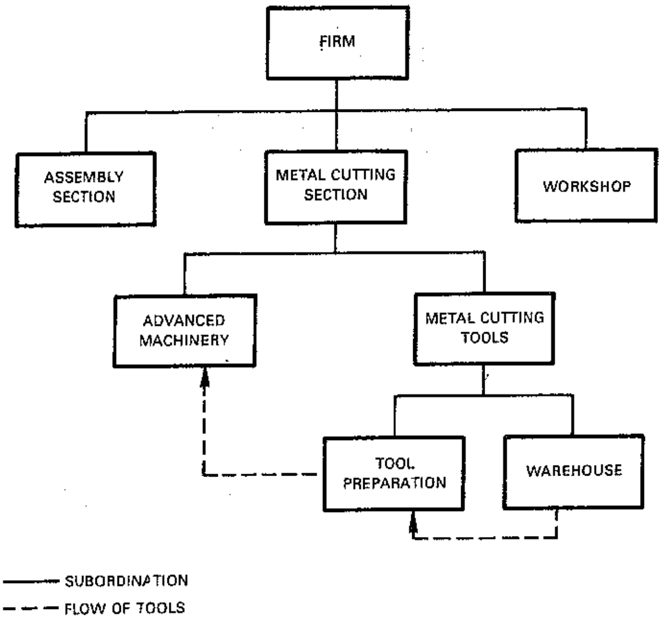

Fig. 1. Organizational structure of the firm.

There are certain minimal requirements of each of the spaces. Space used for storage can be defined by the "net space" which is the actual storage space and by the "gross space" which includes the actual storage area and area to enable man or forklift access. Table 1 presents the net and gross space information for each space category as well as the minimal requirements.

Table 1: Minimal space requirements.

Space category	Net area per unit (m^2)	Gross area per unit (m^2)	Minimal number of units	Minimal area (m^2)
A. Heavy parts	1	6	2	12
B. Superslot	.5	1	25	25
C. Vidmar	.5	1	18	18
D. Office	17	–	–	17

As can be seen from Table 1, an area of 72 m^2 (12 + 25 + 18 + 17) is determined by the minimal requirements. The remaining area of 48 m^2 (120 - 72) has to be allocated among the various categories in a "most" beneficial way.

Obviously, the more we have of each space category, the better. However, we are limited to 48 m^2, and an increase in the allotment to one type of space has to be offset by a decrease in other allotments.

Based on the information of Table 1, the industrial engineers prepared seven alternative area assignments. Four of these plans are presented in Figs. 2 through 5.

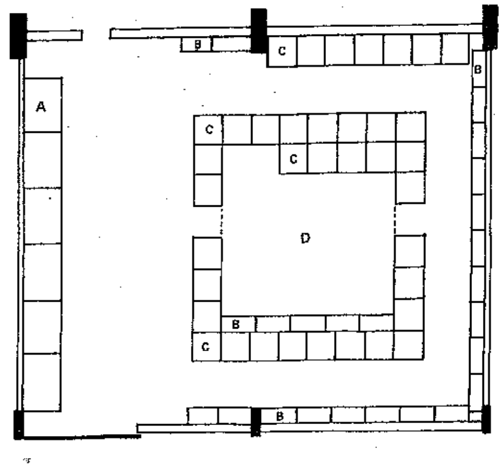

Fig. 2. Assignment # 1.

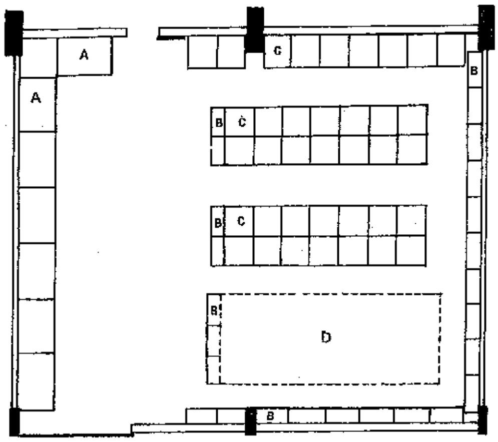

Fig. 3. Assignment #2.

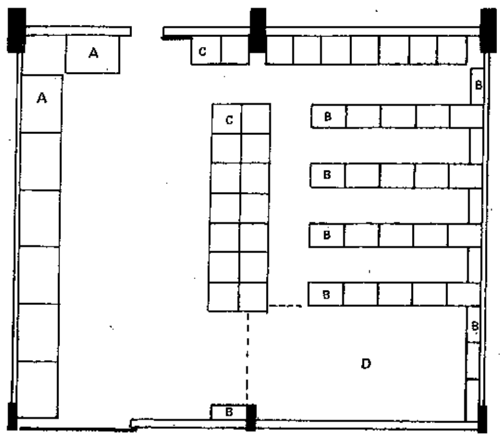

Fig. 4. Assignment # 5.

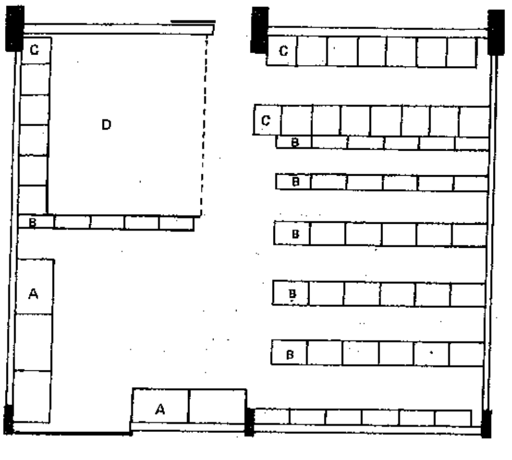

Fig. 5. Assignment # 7.

The seven alternatives differ in how the remaining 48 m^2 were allocated. For example, Alternative 7 has more shelf space (B) than Alternative 1, but has less cabinet storage (C). Assuming dominated alternatives are eliminated, how do we rank the remaining alternatives, taking into account all four space categories? We need tools to handle the multiplicity of evaluators in a systematic manner. The following section will present the methodology and framework needed to rank the alternatives. Again, we are focusing just on space assignment, not design. Thus it is possible to have two identical area assignments presented in different layouts (i.e., arrangements) where one might be preferred over the other. The two areas, however, will have an identical ranking as far as area assignment.

Theoretical Background

The discussion in this section draws heavily on the theory of multiattribute value functions [2].

Let $X_1, \ldots, X_n$ be n evaluators, and assume that each X_i is a monotonically increasing evaluator. Letting each X_i take on a value x_i, we obtain vectors of the form $x = (x_1, \ldots, x_n)$ in a certain evaluation space where we want to observe preferences. We assume: (1) for any two points x' and x'' in the evaluation space, either $x' \geqslant x''$ (to be read: x' is preferred or indifferent to x'') or $x'' \geqslant x'$; if both conditions hold, then we say that $x' \sim x''$ (to be read: x' is indifferent to x''); and if [not $x' \geqslant x''$] holds, we say that $x'' > x'$ (x'' strictly preferred to x'); and (2) that the preference "$\geqslant$" is transitive.

We shall look at points $x = (y, z)$ where y represents those components of x on a previously specified subset of the indices $\{1, \ldots, n\}$, and z represents x on the complementary set of indices. Without loss of generality we can always permute the indices so that $y = (x_1, \ldots, x_s)$ and $z = (x_{s+1}, \ldots, x_n)$. We can view the typical point in the evaluation space as a twotuple (y, z). Naturally, we can also extend this convention to partition the evaluators into two sets $Y = \{X_1, \ldots, X_s\}$ and $Z = \{X_{s+1}, \ldots, X_n\}$.

DEFINITION. y' is *conditionally preferred or indifferent* to y'' given z^* if and only if $(y', z^*) \geqslant (y'', z^*)$. We can therefore describe the conditional preference structure among evaluators Y, given that complementary evaluators are held fixed at z^*.

DEFINITION. The set of attributes Y is *preferentially independent* of the complementary set Z if and only if the conditional preference structure in the y space, given z^*, does not depend on z^*. More symbolically, Y is preferentially independent of Z if and only if $[(y', z^*) \geqslant (y'', z^*)]$ implies $[(y', z) \geqslant (y'', z)]$ for all z, y', y''. In such a case, the decision maker can structure a value function V_Y defined on the y's without specifying a particular z^*.

DEFINITION. The evaluators $X_1, \ldots, X_n$ are *mutually preferentially independent* if every subset Y of these evaluators is preferentially independent of the complementary set of evaluators.

THEOREM. *If every pair of evaluators is preferentially independent of its complementary set, then all the evaluators are mutually preferentially independent.*

The proof of the preceding theorem appears in Keeney and Raiffa [2].

The following theorem is the most important one for our specific application, in conjunction with the theorem stated above. Its proof can be found in the literature [1].

THEOREM. *Let $X_1, \ldots, X_n$ be n evaluators, $n \geqslant 3$. If $\{X_i, X_j\}$ are preferentially independent of the other $n - 2$ evaluators for all i and j, then there exist value functions V_i such that $V(x_1, \ldots, x_n) = \Sigma_{i=1}^n V_i(x_i)$.*

We are faced with the task of assessing a four-attribute value function V(A, B, C, D). The ideal situation, for assessment purposes, is to have a completely additive functional form. If we want to make use of the above theorem, we have to verify that every pair of attributes is preferentially independent of the other two. After carefully evaluating the various combinations, the decision maker reached the conclusion that his preferences over pairs of attributes were independent of the specific levels of the other two attributes. This was verified by considering preferences over all six possible combinations of pairs of attributes while holding the complementary two attributes at fixed levels. These fixed levels were also varied to assure that preferences did not depend on a specific level of any of the complementary attributes. Hence, by the above theorem, an additive value function was justified. In cases involving a very large number of attributes, verifying that every pair of attributes is indeed preferentially independent of the complementary set may become a very tedious task. If we do have complete additivity, the multiattribute value function can be represented as [2]:

$V(x_1, \ldots, x_n) = \Sigma_{i=1}^n \lambda_i V_i(x_i)$ where $\lambda_i \geqslant 0$ for all i, $\Sigma_{i=1}^n \lambda_i = 1$. The λ_k's can be viewed as scaling factors, or coefficients determining the substitution rate among attributes. The assessment of the four-attribute value function thus reduces to assessing four one-attribute value functions and four scaling constants. The following section will discuss the assessment problem, both theoretically and in practice.

Once the one-attribute value functions have been assessed, the scaling constants are calculated by solving simultaneous linear equations or by drawing conclusions from simultaneous inequalities. We have to look for assignments over which the assessor is indifferent. If $x' = (x'_1, \ldots, x'_n)$ and $x'' = (x''_1, \ldots, x''_n)$ are two assignments such that $x' \sim x''$, then $V(x') = V(x'')$ and this implies $\Sigma_{i=1}^{n} \lambda_i V_i(X_i') = \Sigma_{i=1}^{n} \lambda_i V_i(x''_i)$.We need n nonredundant equations to solve for the n λ_i's. It may happen, especially if discrete attributes are involved, that we cannot find enough (or even any) indifferent combinations. We then have to investigate some of the preference relations and observe the associated inequalities between the values from which we can deduce bounds on the λ_i's. We should try to observe enough inequalities to be able to constrict all the λ_i's in narrow enough intervals.

Assessing the One-Attribute Value Functions

Keeney and Raiffa, in their chapter on trade-offs under certainty [2], describe two procedures for assessing an additive value function. We will use here the conjoint scaling technique referred to as the "Midvalue Splitting Technique." In theory, let the range of an attribute X be $x_o \leqslant x \leqslant x_1$, and let Y be another attribute.

DEFINITION. The pair (x_a, x_b) is said to be *differentially value equivalent* to the pair (x_c, x_d), where $x_a < x_b$ and $x_c < x_d$, whenever one is just willing to go from x_b to x_a for a given increase of Y, then one would be just willing to go from x_d to x_c for the same increase in Y.

DEFINITION. For any interval (x_a, x_b) of X, its *midvalue point* x_c is such that the pairs (x_a, x_c) and (x_c, x_b) are differentially value equivalent. We seek a one-attribute value function $V_X(x)$ via the following procedure:

(1) Find the midvalue point of (x_0, x_1); call it $x_{0.5}$ and let $V_X(x_{0.5}) = 0.5$.

(2) Find the midvalue point, $x_{0.75}$, of $(x_{0.5}, x_1)$ and let $V_X(x_{0.75}) = 0.75$.

(3) Find the midvalue point, $x_{0.25}$, of $(x_0, x_{0.5})$ and let $V_X(x_{0.25}) = 0.25$.

(4) As a consistency check, ascertain that $x_{0.5}$ is the midvalue point of $(x_{0.25}, x_{0.75})$.

(5) Fair in the V_X curve passing through points (x_k, k) for $k = 0, 0.25, 0.5, 0.75, 1$.

This procedure carries no conceptual difficulties as long as the attributes are continuous. However, if they are discrete, we cannot be assured of finding any of the midvalue points $x_{0.5}, x_{0.25}, x_{0.75}$. No levels (i.e., values) which the attributes can take may satisfy the requirements for being x_k for some k, $0 \leqslant k \leqslant 1$. Several approaches are available to handle the discrete case [3].

The Actual Assessment

Value Functions

The four attributes in our analysis relate to physical space and are treated as continuous descriptors. There is a slight problem with storage space for heavy parts (A) where the gross area per unit is 6 m^2 and we cannot consider fractional units. Hence only an integer number of units of type A can be considered, ranging from 0 to 8. When we employed the midvalue splitting technique for assessment we did not encounter the need to consider fractional units of type A. In any case, we validated the assessed values using other techniques. Actually, units of Type B and C are also integers, taking on values between 0 and 48, but for all practical purposes those attributes can be considered continuous. Office space (D) is obviously a continuous attribute.

The derived value functions are presented in Figs. 6 through 9. The procedure for assessing the function over the superslot space, V_2(B), is presented in detail. The relevant function is presented in Fig. 7. One must note that all four assessed value functions relate to a space of 48 m^2, which is the only space flexible for different assignments. The initial 72 m^2 are filled by the minimal requirements that are common to all assignments and offer no room for flexibility.

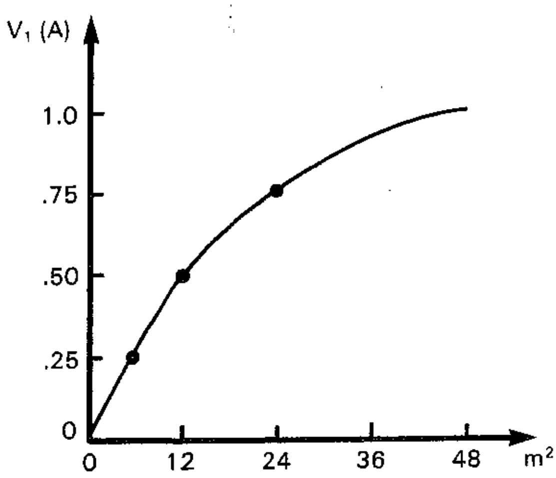

Fig. 6. Value function for heavy storage space (A).

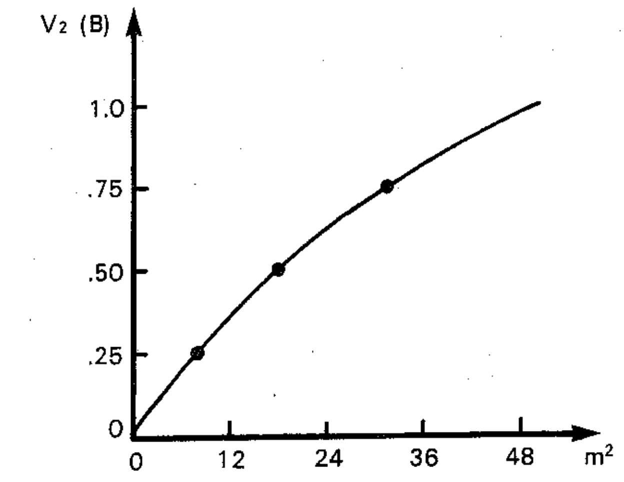

Fig. 7. Value function for shelf storage space (B).

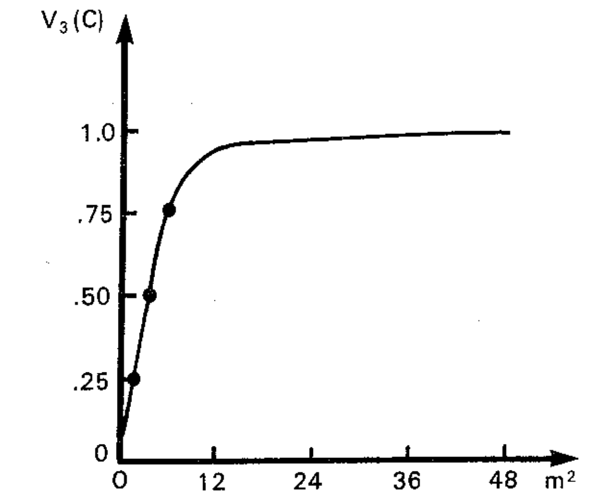

Fig. 8. Value function for cabinet storage space (C).

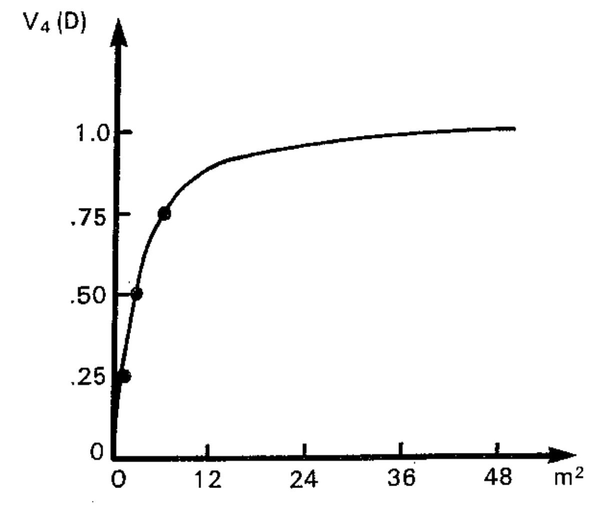

Fig. 9. Value function for office space (D).

Regarding the superslot shelves, the decision maker was first asked the following question: "Suppose you have two warehouses with all minimal requirements (72 m^2) satisfied. One warehouse has no superslot shelves (beyond the minimal requirement of 25) and the other warehouse has an additional 24 m^2 of superslot (for a total of 49 m^2). You can only do one of two things: either increase the superslot space in the first warehouse from 0 to 24 m^2 *or* increase the superslot space of the second warehouse from 24 to 48 m^2. Which would you rather do?" The decision maker exhibited preference for the first warehouse, implying that the midvalue point (whose value is .5 on a 0 - 1 scale) is less than 24. (Or, in other words, the decision valued a shift from 0 to 24 more than a shift from 24 to 48.) When the decision maker valued a shift from 0 to 24 more than a shift of question, he was asked to assess that value of x for which he would be indifferent between increasing the superslot space of the first warehouse from 0 to x m^2 and increasing the superslot space of the second warehouse from x to 48 m^2. The desired value of x is the previously defined midvalue point of the interval (0 - 48). After several rounds and some consistency checks the answer was $x = 18$ m^2, implying $V_2(18) = .5$ [where $V_2(0) = 0$ and $V_2(48) = 1$]. We similarly proceeded with the 0 - 18 range and obtained $V_2(8) = .25$, and the 18 - 48 range where we obtained $V_2(31) = .75$. Fairing in a smooth curve between the five points (0, 0), (8, .25), (18, .5), (31, .75), and (48, 1), we obtain Fig. 7. The other three functions were similarly assessed. Note that the assessment procedure relates only to space and not to the actual layout of this space.

The final forms of the value functions were obtained after several rounds of assessment sessions, where many inconsistencies had to be resolved. Some of the inconsistencies were eliminated by using the same assessment technique (i.e., midvalue splitting), while others required using alternative techniques.

Relative Scaling Constants (weights)

The overall value function is of the form

$$V(x_1, \ldots, x_4) = \sum_{i=1}^{4} \lambda_i V_i(x_i) , \tag{1}$$

where the functions $V_i(x)$ were assessed above. To obtain the relative weights λ_i ($i = 1, \ldots, 4$), we need to establish four nonredundant equations involving the four λ_i's. One requirement (for convenience) is $\Sigma_{i=1}^{4} \lambda_i = 1$; we need three other equations that are obtained from indifference relations between alternative space allocations. The additional available space is 48 m^2, but we did not limit the possible combinations to those exhausting the entire space. Three pairs of equally preferred combinations identified by the decision maker are:

$$(24, 0, 0, 24) \sim (36, 0, 0, 4),$$

$$(24, 24, 0, 0) \sim (36, 0, 0, 0),$$

and

$$(36,0,12,0) \sim (48,0,0,0).$$

By employing Equation (1), these combinations yield, respectively, the following three equations:

$$\lambda_1 V_1(24) + \lambda_4 V_4(24) = \lambda_1 V_1(36) + \lambda_4 V_4(4),$$

$$\lambda_1 V_1(24) + \lambda_2 V_2(24) = \lambda_1 V_1(36),$$

and

$$\lambda_1 V_1(36) + \lambda_3 V_3(12) = \lambda_1 V_1(48).$$

Reading off the values for $V_i(x)$ $(i = 1, \ldots, 4)$ from Figs. 6-9 we obtain

$$.75\lambda_1 + .95\lambda_4 = .9\lambda_1 + .6\lambda_4,$$

$$.75\lambda_1 + .6\lambda_2 = .9\lambda_1,$$

and

$$.9\lambda_1 + .95\lambda_3 = \lambda_1.$$

Together with

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1,$$

the four equations yield $\lambda_1 = .56$, $\lambda_2 = .14$, $\lambda_3 = .06$, and $\lambda_4 = .24$. For any space allocation (A, B, C, D), the value is given by $V(A, B, C, D) = .56V_1(A) + .14V_2(B) + .06V_3(C) + .24V_4(D)$, where the $V_i(x)$ $(x = 1, \ldots, 4)$ are presented in Figs. 6 – 9.

Implementation: Evaluating the Alternative Area Assignments

The value function obtained in the previous section can now be used to score each area assignment and provide a preference ranking for the various alternatives. It should be remembered that the value functions relate to the 48 m^2 that are beyond the 72 m^2 of minimal requirements. Table 2 presents the seven alternative assignments by the number of square meters allocated to each of the four space categories as well as the value (i.e., score) assigned to each alternative by the four-attribute value function.

Table 2: The alternative space assignments (m^2) and their score.

Space category	Alternative 1	2	3	4	5	6	7
A. Heavy parts	24	30	30	30	30	24	18
B. Superslot	0	0	0	1	3	7	16
C. Vidmar	19	17	18	17	5	17	3
D. Office	0	1	0	0	0	0	3
Value (score)	.478	.577	.517	.519	.515	.508	.571

As an example of the calculations, the score for Alternative 4 was obtained as follows:

$$V(\text{Alternative 4}) = V(30, 1, 17, 0) = \lambda_1 V_1(30) + \lambda_2 V_2(1) + \lambda_3 V_3(17) + \lambda_4 V_4(0)$$
$$= (.56)(.82) + (.14)(.02) + (.06)(.96) + (.24)(0) = .5196.$$

The resulting ranking of the seven alternatives (from best to worst) is: 2, 7, 4, 3, 5, 6, 1. Theoretically, we could have stopped here as we achieved our goal and identified assignment 2 as the preferred alternative. However, a closer look at the individual value functions and scaling constants as well as the seven proposed assignments suggested that we can do even better.

Assignment 7 came out a close second, but since it did not utilize all of the available 48 m^2 it was considered for revision. By looking at Figs. 6 - 9, we see that cabinet space (C) and office space (D) offer the most rapid improvement in the value function as long as less than 12 units (of each) are involved.

On the other hand, the scaling constant for D is .24, four times bigger than for C. We also would not ignore the high value put on heavy storage (A), with a coefficient of .56. Therefore, we tried to augment allocation 7 by increasing office or heavy storage space. One suggestion to increase office space resulted in an assignment represented by the vector (18, 11, 4, 8) whose value was .646, already superior to all previous assignments. An alternative improvement involved adding two heavy storage units (A) and decreasing the number of shelf storage units (B) by four (depicted in Fig. 10). It can be represented by the vector (30, 12, 3, 3) whose value is .654. This last space assignment was then recommended and accepted by management.

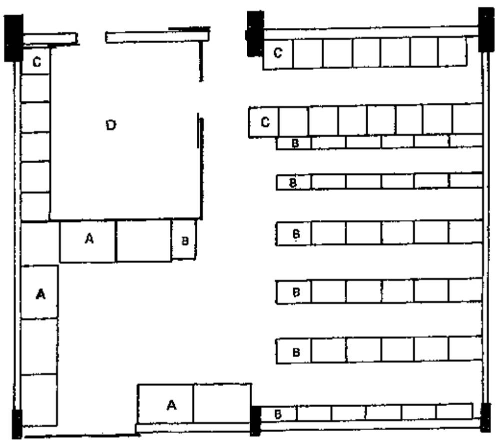

Fig. 10. Final space assignment.

This process of refinement would have been very difficult to achieve without the formal analysis. True, it was known that heavy storage space was more valuable than other space, but we did not know the relative magnitude of importance. We also did not explicitly know the marginal benefit to be derived from increments within each space category. The formal analysis of explicitly assessing the individual value functions and trade-offs among the four competing activities enabled us not only to rank order the alternatives but also provided important insight into the design of the actual layouts.

We have validated the assessed value function by comparing several "simple" area assignments involving only two of the four space categories. (The decision maker was asked to rank those alternatives.) We then applied the value function to those simple assignments and compared the ranking yielded by the function to the "manual" ranking. The two rankings were in complete agreement.

Discussion

It has just been demonstrated that, given certain assumptions, we can rank order area assignments to be consistent with a decision maker's perception and preferences. Here it is appropriate to discuss those "certain assumptions." As a matter of fact, only one assumption, mutual preferential independence of all space attributes, has been invoked, and it is claimed to approximate reality. We have actually invoked a seemingly "weaker" assumption that every pair of attributes is preferentially independent of the remaining two. However, we have stated a theorem where this assumption implies mutual preferential independence. It is much easier in practice to verify whether pairs of attributes are independent of the subset of remaining attributes than it is to verify mutual preferential independence. This verification process may, however, become very laborious as the number of attributes increases.

We have already seen how preferential independence is interpreted mathematically. Let us now examine how it can be interpreted in the specific context of our problem involving space attributes in an allocation. Mutual preferential independence really indicates a strictly additive contribution of each attribute to the total satisfaction from a given area assignment. The effect of all attributes on the overall score of an assignment is the simple sum of individual effects. In other words, no interaction of any kind occurs between any two attributes. Therefore, if two attributes are present simultaneously, each contributes its own effect. No effect (positive or negative) flows from simultaneous presence of both space categories. Preferences over one space category are unaffected by the space allocated to another category.

How "real" is the additivity assumption in general? Many phenomena, consisting of various components, can in fact be evaluated through an additive model of individual components. Even if the real situation is not additive, an additive representation may well serve the relevant purpose. In a multiattribute situation, people often tend to evaluate their preferences for multiattribute factors by thinking in terms of additive effects of each attribute. If their thinking and perception tend to be additive, an additive assessment scheme may well serve the purposes. In many instances, including interaction of nonadditive effects provides little more insight to various evaluations. If there is no obvious reason to doubt mutual preferential independence, we should proceed on the premise that additivity does in fact hold.

There are obvious cases where interaction cannot be ignored. In our specific application of warehouse area assignments, let us consider only the net area of each space category and introduce a fifth space attribute, E, which designates total forklift access space. If we now consider, for example, the preference structure over attributes A and B while holding the other three attributes at fixed levels, we may find that this structure depends on the specific level of access space (E). If there is no access space then increasing space A will not necessarily increase the value of a given allocation because the additional space cannot be used because of lack of access. As a matter of fact, additional unusable space of a given type may actually decrease the value of a given allocation.

Sometimes it is possible to handle such situations by combining preferentially dependent attributes into single, more complex attributes. If space type A is preferentially dependent on space type E, we can consider an attribute AE, and hopefully the four attributes, AE, B, C, D will be mutually preferentially independent. This is what we actually did, although we achieved this by originally considering the gross space of each attribute as opposed to the net space. We simply incorporated the needed access space into the space definition.

There will be cases where the additivity assumption cannot be "reconstructed." As an example, consider a warehouse that has only three different storage methods; namely, tote storage, bin shelving, and pallet storage. In real life situations, every item carried in stock will have its "preferred" storage method based on the activity and cube of the item. When the item is received it will be stored in its "preferred" method if storage space is available. If storage space under the "preferred" method is not available, the item will be stored under the "next preferred" method, and so on, until the item is assigned a storage location. Now, let us assume that all the items with preferred bin shelving storage will be stored in tote storage if sufficient space is not available in bin shelving. Under such conditions, how much space the decision maker is willing to transfer from bin shelving to pallet storage will be a function of how much space he is provided with in tote storage.

In our evaluation of area assignment it always makes sense to use up all the available space of 48 m^2 (if not, the total value can always be increased by increasing any of the spaces so that the total equals 48 m^2). This situation totally ignores costs. Actually, costs could be considered as a separate attribute and then it may be possible that a preferred assignment will not involve the entire available space, simply because the increased contribution of the additional space will be offset by the high costs of using this space. The decision makers in our analysis were not concerned with costs when evaluating the proposed assignments, so this attribute was not explicitly introduced.

As mentioned earlier, we have focused just on the space allocation problem. Two layout designs having the same space allocated to each category will not necessarily be equally desirable to management. One could be aesthetically nicer (e.g., more symmetric) or more functional (e.g., proximity of one space category to another). These considerations are important and could perhaps be incorporated into the analysis by including such additional attributes as material flow between each pair of departments and its associated unit cost. We chose not to, simply because we were presented with seven alternative space assignments already not in tabular form but as architectural designs of layouts, and were told to evaluate them solely on the basis of space allocation. Naturally, the actual layout design could be a totally different issue but it would be difficult to separate it from an area assignment.

In an evaluation process such as the one discussed in this analysis, it may be helpful to assess the multiattribute value function before the initial design phase. The formal assessment process and the explicit consideration of trade-offs could provide management and the industrial engineers with much needed insight and guide them through the assignment and design phase to come up with superior space allocations.

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